

18. If G is a K -regular bipartite graph, prove that G has a perfect matching.

19. Prove that $r(K, l) \leq \binom{K+l-2}{K-1}$.

20. State and prove Euler's theorem for plane graph.

NOVEMBER/DECEMBER 2024

23PEMA14B — GRAPH THEORY AND APPLICATIONS

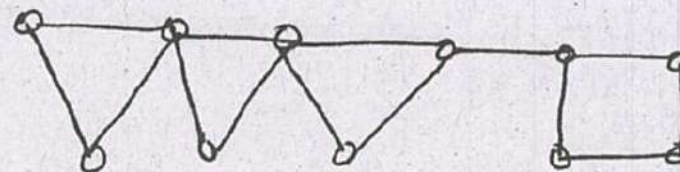
Time : Three hours

Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define degree of a vertex. What is the degree of K_n and C_n ?
2. Define cut edge and cut vertex of a graph.
3. State Menger's theorem.
4. State Whitney theorem.
5. Find maximum matching and minimum covering numbers of



6. State Vizing's theorem.

7. Define Ramsey number of a graph.
8. State Brook's theorem.
9. Prove that $K_{3,3}$ is non-planar.
10. State five colour theorem for planar graph.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Show that the number of odd degree vertices of G is even.

Or

- (b) Show that $\epsilon(G) = \gamma(G) - 1$, if G is a tree.

12. (a) Prove that G is Hamiltonian $\Leftrightarrow$ its closure is Hamiltonian.

Or

- (b) Give an examples for $K < K' < \delta$ and $K' = K = \delta$.

13. (a) What is the difference between maximum and maximal matching of G . Give an example.

Or

- (b) Let M be a matching and K be a covering such that $|M| = |K|$, prove that M is maximum and K is minimum.

14. (a) Prove that in a critical graph has no cut vertex is a clique.

Or

- (b) Show that $\alpha' + \beta' = \gamma$.

15. (a) Prove that $\delta(G) \leq 5$, G is a simple planar graph.

Or

- (b) Show that K_5 is non-planar.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that if e is a cut edge of G , then e is in no cycle of G .
17. If G is a simple with $\gamma \geq 3$ and $\delta \geq \frac{\gamma}{2}$ show that G is Hamiltonian.